The long-run success of mutual funds

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Abstract

This paper investigates the long-run nature of a market populated by heterogeneous asset managers that must compete for investment capital. For market conditions similar to those of U.S. equity markets the optimal amount of portfolio riskiness is computed if a manager desires to maximize the average assets under management of a fund. The framework also allows for the study of the distribution of assets across investment companies. It is demonstrated that in conditions similar to those of U.S. equity markets maximizing a fund's average AUM entails holding significantly more risk than that of the market portfolio.

*This is a preliminary draft. Please do not cite without permission. I am grateful to David Easley, Carlos Alos-Ferrer, Bryan Ricchetti, Joseph Podwol and Hwan-sik Choi for helpful comments. **Address for Correspondence:** Scott Condie, Department of Economics, Brigham Young University, Provo, UT 84602. E-mail: scott_condie@byu.edu

1 Introduction

The value of equity and bond market assets that are under active management is growing. While much empirical evidence indicates that returns from actively managed investment funds do not on average outpace those of broad market indices, this important sector of global markets is too large to ignore. This paper incorporates some of the important features of financial intermediaries who engage in active management into a traditional asset pricing model and analyzes the intertemporal nature of equilibrium in this market.

Traditional asset pricing models are useful for making predictions about the long-run viability of particular trading strategies. In such models, the amount of market capital available for investment at any given time for a market participant is made up of the market returns garnered by that participant to that point less any previously realized personal consumption. In such models, it has long been surmised (see Alchian (1950) for an early example) that some trading strategies in the market would have better chances of long-run survival than others. Some examples of this concept include Sandroni (2000) who demonstrates that if markets are complete then in the long-run a trader that has correct beliefs about the relative probabilities of future market payoffs will, by making wise bets against less knowledgeable market participants, earn returns that strip investment capital from less knowledgable traders and eventually drive these traders from the market.

A market that is populated by investment companies differs from the markets populated by long-lived utility maximizing investors in at least two respects.

The first is that although as in other markets, the amount of investment capital available to a company at any time is positively related to previous returns, the mechanism through which this happens is indirect. It is the investors in the company that ultimatly have claim to the returns that have been garnered. These investors have investment goals and liquidity needs that change in a random way and that subject the company to changes in available investment capital that are more difficult to forecast. Secondly, investment companies are under contract to act in particular ways with the funds that have been invested. For some funds this contract is quite explicit (e.g. index mutual funds are charged with mimicking the behavior of a particular index). For others this contract may be more broadly defined (e.g. earn returns that are higher than those of the S&P 500). Given these observations, it is not clear that the behavior of an investment company will mimick that of a long-lived individual investor.

The goal of this paper is to study the properties of a market that incorporates some of these deviations from more traditional asset pricing models. This will shed light on the types of investment strategies one should see thriving in the market and the distribution of investment capital across such strategies. It will be demonstrated that: 1) except in exceptional circumstances wealth will not become concentrated in one particular investment company as other models suggest, 2) whether a strategy performs well over time (in a sense to be defined later) depends on several factors including the other strategies in use in the market and the properties of the relationship between returns that are garnered by investment companies and the subsequent inflow and outflow of new investment capital. Furthermore, for some calibrations of the market environment the optimal (in terms of average AUM) riskiness of a fund's portfolio can be calculated.

1.1 Related Literature

The literature on mutual fund behavior is vast. While significant work has been done to empirically estimate the nature of the return-flow relationship (e.g. Sirri and Tufano (1998) and Chevalier and Ellison (1997)), much less work has been done on the theoretical implications of this result to asset prices, mutual fund survival and other empirically observable market data. A notable exception to this is the work of Berk and Green (2007). Their work builds a model of manager talent and derives the optimal contract that investors should employ. The market environment in which managers operate is modelled through partial equilibrium. This paper on the other hand, takes a partial equilibrium approach to the contract that is offered to fund managers. This return-flow relationship is exogenous to the model. However, equilibrium in the asset market in which investment companies operate is modelled explicitly and prices as well as returns are dependent on the behavior of all market participants.

The questions that I ask and the methods that I employ in answering them draw from earlier work on market selection and the survival of investment strategies. Although the statement of the market selection hypothesis seems to date back to Alchian (1950), modern work on this question was begun in Blume and Easley (1992). The model used in this paper was inspired by, but is distinct from the model given in Alos-Ferrer and Ania (2005).

Blume and Easley (1992) analyze the survival of traders in a market like the market that I consider, and find that across constant investment strategies and if all market participants have identical savings rates, there is one strategy (betting correct beliefs) that drives out all other strategies. Their analysis, like mine does not depend on utility maximizing behavior. Sandroni (2000) provides the important result that in a complete market populated by utility maximizing individuals with identical discount factors, individuals with correct beliefs drive out individuals with incorrect beliefs almost surely. This result is expanded by Blume and Easley (2006) where it is shown that in all Pareto optimal outcomes agents with correct beliefs survive.

In this model however, I give conditions under which the the noise in the market selection mechanism that is introduced when investors use intermediaries for investment is sufficient to cause these market selection results to not hold. The essential assumptions for this result are the two assumption contained in the next section.

2 The Model

Time is infinite and discrete and indexed by $t \in \mathbb{N}$. Let $\Omega = \{s_1, s_2, \ldots, s_N\}^\infty$ be the state space with generic element ω. Let $\mathcal{C}_t(\omega)$ be the set of paths whose first t elements agree with the first t elements of ω Let P be the distribution over $(\Omega, \mathcal{F})$ At each date t there exists an identical set of securities that span the space of possible state realizations next period. The security labelled n is assumed to payoff gross return R_n if the state s_n is realized and zero otherwise. Each security is assumed to have supply of one.[1]

Traders in the market will be called mutual funds and there are I of them. These mutual funds compete for investment capital. To fix intuition, one might think of these funds as a set of small cap mutual funds who invest in small cap stocks, are benchmarked against a common benchmark and whose performance is compared when investors are deciding on a small cap fund.

Mutual funds receive capital for investment from a single outside investor who has no direct access to the security market. For simplicity it is assume that at the end of each period ownership of all assets is passed back to the investor who then decides how to redistribute this investment capital, plus any additional capital that is to be invested amongst the funds.

In each period t, the following sequence of events transpires:

1. Payoffs from the previous period are distributed to the investor.

2. The investor decides where she will invest for next period.

3. Funds make investment decisions.

4. Equilibrium prices are determined in the asset market .

5. The state is realized and payoffs/returns are calculated.

[1]So the payoff R_n represents the payoff in the market over the period. Many equivalent asset structures exist that will mimic the payoff structure given here and analysis with any of these is, by the market clearing requirement of equilibrium, equivalent. However, this asset structure is chosen for the tractability that it provides.

To concentrate on the issues at hand it is assumed that the investor does not consume any of her realized profits and that mutual funds provide their investment services at no commission. These simplifications allow for a better understanding of the role that differential strategies play in fund success.

2.1 Mutual funds and the market investor

Funds must decide how to allocate their available investment capital (labelled assets under management or AUM throughtout) amongst the assets that are available to them. In any period t, fund i selects a portfolio strategy $\alpha^i = \alpha_1^i, \ldots, \alpha_N^i$ that designates the fraction of the fund's available investment capital that will be used to purchase each of the N assets. It is assumed that $\sum_{n=1}^{N} \alpha_n^i = 1$.

At any period t, if each of the investment firms has capital w_t^i and follows the strategy α^i then the price of the asset paying off in state n at time $t+1$ is

$$p_n(t) = \sum_{i=1}^{I} \alpha_n^i w^i \tag{1}$$

The gross return at time t to a particular strategy α is given by the random variable

$$r_t(\alpha) = \left(\frac{\alpha_1}{p_{t1}} R_1\right)^{\mathbf{1}_t(s_1)} \left(\frac{\alpha_2}{p_{t2}} R_2\right)^{\mathbf{1}_t(s_2)} \cdots \left(\frac{\alpha_N}{p_{tN}} R_N\right)^{\mathbf{1}_t(s_N)} \tag{2}$$

where $\mathbf{1}_t(x)$ is the indicator function that is 1 when $\omega(t) = x$.

The market investor observes realized returns and makes the decision about whether to increase, decrease or hold constant the amount of capital that will remain with the fund to invest. This decision is made according to the function $P : \mathbb{R}^I \rightarrow (\Delta^2)^I$ that takes as an argument a set of returns $r^1, r^2, \ldots, r^I$ for each fund and returns a probability of increasing, decreasing or leaving unchanged the amount of investment capital that each fund will manage. It is assumed that the AUM that can be allocated to each fund comes from a finite grid with elements $w_0, \ldots, w_\phi, \ldots, w_\Phi$.

The returns r^i are defined in (2) and depend on both strategies and prices, both of which may depend on the previous wealth levels of all of the funds.

Denote the profile of fund wealths at time t defined by $(w^1_{\phi^1}, \ldots, w^I_{\phi^I})$ as the profile $\phi = \{\phi^1, \ldots, \phi^I\}$. The probability that the AUM profile of funds moves from ϕ to some other profile ϕ' is given by $P_{\phi,\phi'}$. This probability will be determined by the difference between the funds performance and a baseline return.

Thus, if r^i is the random variable representing the return to fund i and r^f is the baseline return, then $P_{\phi,\phi'} = f(r^1 - r^f, \ldots, r^I - r^f)$. The terms $P_{\phi,\phi'}$ define a probability transition matrix between the possible AUM profiles that can be observed in the market.

Assumption 1 *Let Δ be an I-tuple of elements of the set $\{-1, 0, 1\}$. For any ϕ, $P_{\phi,\phi+\Delta} > 0$ for all r.*

Assumption 2 *The strategy α^i of fund i is a deterministic function of asset prices p current investment capital w^i.*[2]

[2] *This assumption can be weakened to allow for strategies that depend on a finite past*

The function $f(r)$, gives the probability of moving from state ϕ to state ϕ' conditional on the returns r being realized. If R is the set of returns which can feasibly be realized next period given current period strategies and prices and π is the probability distribution over states next period then the probability of going from state ϕ to state ϕ' is given by

$$E_r P_{\phi,\phi'} = \sum_{\omega(t+1)\in C_{t+1}(\omega)} \pi(s) P_{\phi,\phi+\Delta}(r(\omega(t+1))) \tag{3}$$

Assume that 1 and 2 hold. By assumption 2 we have that there exists a finite set of possible returns that can be realized. To see this, suppose that there were an infinite set of return profiles that could be observed by the market investor. Let $\mathcal{S} = \{(\phi, r_T)$ where ϕ is a distribution of capital across the funds and r_T is a profile of previously earned T-period returns across the

Proposition 1 *Given assumptions 1 and 2, there exists a unique invariant distribution over the set of wealth distributions across funds. Furthermore the empirically observed intertemporal average capital distribution across funds will converge to this invariant distribution. Lastly, this distribution places positive weight on all possible AUM profiles.*

Proof. This result follows from the fact that the transition between wealth states can be written as a first-order Markov process with transition matrix P that is irreducible, aperiodic and positive recurrent. To see this, let ϕ be an AUM

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history of returns, prices and AUM profiles. However, the improvement in the results of the paper is not large enough to outway the increase in complexity that such generalities entail.

profile across market participants. Assumption 1 implies that the transition between AUM states ϕ and ϕ' is strictly positive if $\max_i(\phi^i - \phi^{i\prime}) \leq 1$. From this it follows that the transition matrix P is irreducible. This fact, combined with the finiteness of the state space implies that every AUM profile is positive recurrent and that P is aperiodic.

The statement that the empirical observations of the AUM distribution converge to the invariant distribution is a standard result in Markov Chains (again see Norris (1997)[Sections 1.7 and 1.8]). In particular, for any AUM profile ϕ let π_ϕ be the probability of ϕ occuring under the invariant distribution and let $\phi(t)$ be the AUM profile that arises in period t. Then

$$\pi_\phi = \lim_{t\to\infty} \frac{1}{t}\mathbf{1}(\phi(t) = \phi) \text{ a.s.} \tag{4}$$

That the invariant distribution places positive weight on each possible AUM profile follows from the detailed balance conditions that will hold for an invariant distribution with a positive recurrent, aperiodic transition matrix of the form given here (see Norris (1997)[Section 1.9]). These state that if π is the invariant distribution and ϕ and ϕ' are two AUM profiles then

$$\pi_\phi P_{\phi,\phi'} = \pi_{\phi'} P_{\phi',\phi} \tag{5}$$

Notice that if $\max_i(\phi_i - \phi'_i) \leq 1)$ then both $P_{\phi,\phi'}$ and $P_{\phi',\phi}$ will be non-zero.

If $\pi_\phi = 0$, then by equation (5) $\pi_{\phi'} = 0$. One may then show that $\pi_{\phi''} = 0$ for all ϕ'' by relating all AUM profiles to π_ϕ in the way just done.. ■

This result provides information on the distribution of observed AUM profiles in the market. One may want to summarize such distributions by means of particular moments that can be calculated from them. For example, it would be interesting to know whether the observed intertemporal average AUM of fund i is greater than that of fund j.

To understand how to answer this question, let $W = \{w \in \{w_1, \ldots, w_\Phi\}^I\}$ be the set of all possible wealth distributions across investment companies. By definition,

$$E_\pi w^i = \sum_{w \in W} \pi(w) w^i \tag{6}$$

and

$$E_\pi w^i - E_\pi w^j = \sum_{w \in W} \pi(w) w^i - \sum_{w \in W} \pi(w) w^j \tag{7}$$

Solving for this invariant distribution and analyzing the properties of the distribution is not always analytically possible. The invariant distribution is by definition the eigenvector associated with the unit eigenvalue of the transition matrix P. Thus the invariant distribution is defined implicitly by the equation $\pi P = \pi$. Furthermore, the matrix P is a somewhat complicated function of each individual company's investment strategy and the market investor's decision rule. Depending on the number of wealth states that are considered, the complexity of investment strategies and the market investor's decision rule, this invariant distribution can be quite difficult to analyze.

However, some results can be obtained for a few interesting cases. One of these examples is taken up next. Section 3 investigates the empirical properties of this invariant distribution.

2.2 A competitive market

There are two states of wealth w_1 and w_2 and consequently four possible AUM profiles. The market is composed of one investment company (1) that holds a risk-neutral portfolio denoted by α^1 and another company (2) that uses a possibly different investment strategy α^2 . The set of states $(s_1, \ldots, s_N)$ have probabilities $(q_1, \ldots, q_N)$ and total payoffs $(R_1, \ldots, R_N)$. The investment rule that the market investor uses is allowed to be assymetric as in **??** since the relative increase in fund flows as a result of high returns (given by b_i below) can be different from the relative decrease in fund flows (b_d below) from low returns. It is given by

$$\begin{aligned} P(w\downarrow, w, w\uparrow) &= (P_1, P_2, P_3) \\ &= (a + b_d(r-\bar{r})^{\mathbf{1}(r<\bar{r})}, 1 - P_1 - P_3, a + b_i(r-\bar{r})^{\mathbf{1}(r>\bar{r})}) \end{aligned} \tag{8}$$

where $w\downarrow$ indicates the event that AUM decreases and $w\uparrow$ is the event that AUM increases. The return in each state for company i is given by (2) and is labelled r^i. Consequently, the expected value of this transition rule (before returns are realized) is given by

$$\begin{aligned} EP(w\downarrow) &= a + b_d \sum_{n:r_n^i<\bar{r}} q_n(r_n^i - \bar{r}) \\ EP(w) &= 1 - P_1 - P_3 \\ EP(w\uparrow) &= a + b_i \sum_{n:r_n^i>\bar{r}} q_n(r_n^i - \bar{r})) \end{aligned} \tag{9}$$

Substituting in the strategies of company 2 shows that this company's probabilities of increasing or decreasing its current assets under management are

$$\begin{aligned} EP(w\downarrow) &= a + b_d \sum_{n:r_n^i<\bar{r}} q_n \left(\frac{\alpha_n^2 R_n}{p_n} - \bar{r} \right) \\ EP(w) &= 1 - P_1 - P_3 \\ EP(w\uparrow) &= a + b_i \sum_{n:r_n^i>\bar{r}} q_n \left(\frac{\alpha_n^2 R_n}{p_n} - \bar{r} \right) \end{aligned} \tag{10}$$

Finally, since by assumption the rest of the market (company 1) holds a portfolio that corresponds to a risk neutral strategy, in equilibrium the expected return of each asset $q_n R_n / p_n$ will be the same for all assets. Labelling this expected return $E(R)$ and simplifying produces

$$\begin{aligned} EP(w\downarrow) &= a + b_d E(R) \sum_{n:r_n^i<\bar{r}} (\alpha_n^2 - q_n \bar{r}) \\ EP(w) &= 1 - P_1 - P_3 \\ EP(w\uparrow) &= a + b_i E(R) \sum_{n:r_n^i>\bar{r}} (\alpha_n^2 - q_n \bar{r}) \end{aligned} \tag{11}$$

Denote the change in AUM profiles of funds 1 and 2 by (w^1, w^2). So

$(w^1, w^2) = (w_2, w_1)$ means that fund 1 is in the high AUM state and fund 2 is in the low AUM state. Suppose that company 2 wants to select its strategy so that its long-run average AUM is greater than that of company 1. Then it must select its strategy so that

$$\begin{aligned}\sum_{w \in W} \pi(w)(w^2 - w^1) &= \sum_{i_1=1}^{2} \sum_{i_2=1}^{2} \pi(w_{i_1}, w_{i_2})(w_{i_2} - w_{i_1}) \\ &= \pi(w_1, w_2)w_2 + \pi(w_2, w_1)w_1 - \pi(w_1, w_2)w_1 - \pi(w_2, w_1)w_1 \\ &= (\pi(w_1, w_2) - \pi(w_2, w_1))(w_2 - w_1) > 0\end{aligned} \tag{12}$$

Since $w_2 - w_1 > 0$ by definition, the set of strategies that lead to a higher long-run average AUM for company 2 is that set of strategies for which $\pi(w_1, w_2) - \pi(w_2, w_1) > 0$.

This set can be characterized by considering the detailed balance equations for the the invariant distribution π. These say that

$$\pi(w_1, w_2)P((w_1, w_2), (w_2, w_1)) = P((w_2, w_1), (w_1, w_2))\pi(w_2, w_1). \tag{13}$$

From this it can be seen that company 2 will have a larger long-run average AUM than company 1 if

$$Q = \frac{P((w_2, w_1), (w_1, w_2))}{P((w_1, w_2), (w_2, w_1))} > 1 \tag{14}$$

That is, the equilibrium probability of going from the profile (w_2, w_1) to (w_1, w_2) is greater than the equilibrium probability of going from (w_1, w_2) to

(w_2, w_1).

Using the market investor's investment rule given in (11) this can be written as

$$Q = \frac{(a + b_d E(R) \sum_{n:r_n^1<\bar{r}}(\alpha_n^1 - q_n\bar{r}))(a + b_i E(R) \sum_{n:r_n^2>\bar{r}}(\alpha_n^2 - q_n\bar{r}))}{(a + b_i E(R) \sum_{n:r_n^1>\bar{r}}(\alpha_n^1 - q_n\bar{r}))(a + b_d E(R) \sum_{n:r_n^2<\bar{r}}(\alpha_n^2 - q_n\bar{r}))} \quad (15)$$

Thus, if $Q > 1$ then company 2 will have larger long-run average AUM than company 1. The opposite is true if $Q < 1$.

In order to understand Q, there are several cases that must be considered. First, if the benchmark is sufficiently low that in both states both firms will beat the benchmark then $Q = 1$ since $\sum_{n:r_n^1<\bar{r}}(\alpha_n^i - q_n\bar{r}) = 0$ for both companies and $\sum_{n:r_n^2>\bar{r}}(\alpha_n^2 - q_n\bar{r}) = 1 - \bar{r}$ for both companies. In this case the long-run average AUM of each company will be equal. For analogous reasons, if the benchmark is sufficiently high that with probability one both firms will miss the benchmark, then average AUM across companies will also be equal.

However, the distributions implied by these two cases will be different in general. If the benchmark is low then it can be shown that the average asset holdings of each company are higher than if the benchmark were high. This is shown by using the detailed balance conditions to investigate the relative sizes of $\pi(w_k, w_1)$ vs. $\pi(w_k, w_2)$ and $\pi(w_1, w_k)$ vs. $\pi(w_2, w_k)$ for any k.[3]

Another interesting case occurs if the benchmark $\bar{r}$ has the possibility of distinguishing between the two companies. One example of such a benchmark

[3] As this is straightforward, I omit the demonstration.

is a market-based benchmark like the returns to a given market portfolio (such as the $S\&P500$). In this model, where only investment companies trade and thus determine prices, if the market in which traders interact consists of all $S\&P500$ stocks then when one trader beats the market return, there must be another trader whose return is lower than the market return.

In this case the condition $Q > 1$ can be rearranged to be equivalent to the condition

$$(ab_i + ab_d + b_d b_i E(R)(1 - \bar{r}))(\alpha_n^1 + \alpha_m^2 - q_m \bar{r} - q_n \bar{r} - 1 + \bar{r}) > 0 \qquad (16)$$

Where m is the state in which company 2 beats the benchmark and n is the state in which company 1 beats the benchmark. If the benchmark is taken to be the market return, then $q_m + q_n = 1$, since both companies must beat the benchmark in separate states.

In this situation, company 2 will have a higher observed average AUM if

$$\alpha_n^1 + \alpha_m^2 - 1 > 0 \qquad (17)$$

Since we have assumed that the market outside of firm 2 (firm 1 here) uses a risk-neutral portfolio strategy, it must be true that

$$\frac{\pi R_1}{p_1} = \frac{(1-\pi) R_2}{p_2}. \qquad (18)$$

The market clearing equations for prices at any period are that $p_1 = w^1 \alpha^1 + w^2 \alpha^2$

and $p_2 = w^1(1-\alpha^1) + w^2(1-\alpha^2)$. Substituting these equations in to the condition (18) and rearranging provides

$$\alpha^1 w^1 + \alpha^2 w^2 = \frac{\pi R_1}{\pi R_1 + (1-\pi)R_2} w^1 + \frac{\pi R_1}{\pi R_1 + (1-\pi)R_2} w^2. \tag{19}$$

In this market with one very active arbitrageur (company 1), the above equation defines the portfolio rule of company one in terms of the behavior of company 2. Plugging this implied rule into the requirement (17) and simplifying gives that company 2 will have higher average AUM than company 1 if

$$\alpha^2 > \frac{\pi R_1}{\pi R_1 + (1-\pi)R_2} \text{ or } \alpha^2 < \frac{\pi R_1}{\pi R_1 + (1-\pi)R_2}. \tag{20}$$

That is, if company 2 employs any strategy that is different than the equilibrium strategy that it would employ if both firms were risk neutral. The market implications of this result are interesting. What it demonstrates is that in a market in which arbitrageurs will determine prices, an investment company need only differentiate itself in some way in order to have higher long-run average AUM than the arbitrageur. This happens because the risk neutral mutual fund bears the burden of ensuring that expected returns are equal. Thus, the other fund can trade without offering any price concessions. This allows the second fund to earn the higher expected returns that come from differentiation.

3 Mutual fund success in equity markets

This section provides the results of several attempts at simulating the conditions that an ivestment company may experience in currently operating markets and the related results on the success of strategies encompassing different risk levels. To parametrize the riskiness of portfolios decision rules, I consider portfolios generated by companies following constant relative risk aversion generated strategies. This class of strategies is convenient because the coefficient of relative risk aversion is a single parameter that appropriately measures the riskiness inherent in the portfolio. Furthermore, this class of strategies suggest a relatively low and stable risk-free rate, which has been observed in U.S. equity and fixed-income markets.

Literature on the equity premium puzzle suggests that in order to accurately represent the premium to equity in U.S. markets with a CRRA investor and no market frictions, this investor would need to have a coefficient of relative risk aversion greater than 20. To synthesize market conditions similar to those that are currently observed, all of the results presented below will compare the relative success of an investment company that participates in the market using a given risky strategy with another market participant that is assumed to follow a CRRA strategy with a coefficient of 30. Changes in this parameter have quantitative effects on some of the results presented but do not appear to make a qualititative difference.

For each of the simulations presented below I hold the investment rule of the market investor constant. The responsiveness of capital inflows and outflows to performance in accordance with recent literature (see ?? and ??). The rule in use throughout this section is that given by equation (11). For these examples the parameter $a = 0.1$, while $b_d = 0.1$ and $b_i = 0.3$. Thus, the return flow relationship is assymetric with positive returns having three times the effect on inflow that negative returns have on outflow. In an attempt to match the empirical work that has been done previously in Sirri and Tufano (1998) and Chevalier and Ellison (1997), the benchmark is taken to be 3 percent (approximately the risk free rate).

Figure 3 demonstrates the shape of the AUM distribution when company 1 and company 2 hold portfolios with equal risk. The symmetry in the distribution of AUM across funds is apparent from the diagram. The distribution demonstrates several properties that are commonly discussed in the institutional literature. First, it is noted that investment opportunities diminish as the AUM of each fund increases. It has often been asserted that at high levels, AUM is negatively correlated with expected returns. In these examples this occurs because as funds become large, taking asset positions requires ever larger price movements to accomodate the new position. These price movements decrease expected returns. Eventually investors' required rate of return, which is implicit in the investment decision rule, limits the AUM of each management company. For analogous reasons, each company's AUM is not concentrated at zero because with relatively low AUM, price impact is quite small and AUM is low, leading to assets that are relatively underpriced.

Although the distribution of AUM across portfolios can be computed ex-

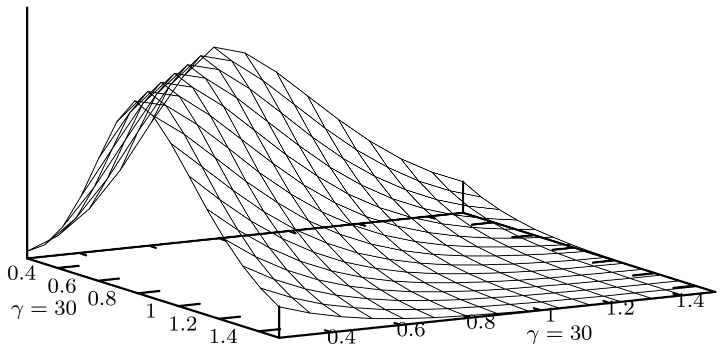

Figure 1: The distribution of AUM across two investment companies with identical strategies.

plicitly, some moments of these distributions are interesting to analyze. Figure 3, represents the calculation of one such moment of the distribution of assets across firms. In figure 3 I plot the expected value, or intertemporal average of AUM over time for two companies. One of the companies is assumed to select portfolios bases on a CRRA coefficient of 30. The other company's CRRA coefficient is plotted on the horizontal axis. It can be seen that with the investment rule used in this example, the wealth share of the investment company with a variable CRRA coefficient is strictly decreasing in the coefficient of relative risk aversion over the range $\gamma \in [5, 30]$ and is increasing over the range $\gamma \in [2, 5]$.

This plot demonstrates that in the market constructed here, if the goal of a particular fund is to increase the long-run average AUM of the fund, then in the range presented the company has every incentive to increase the riskiness of the portfolio when faced with a market that has behaves as if it has a coefficient

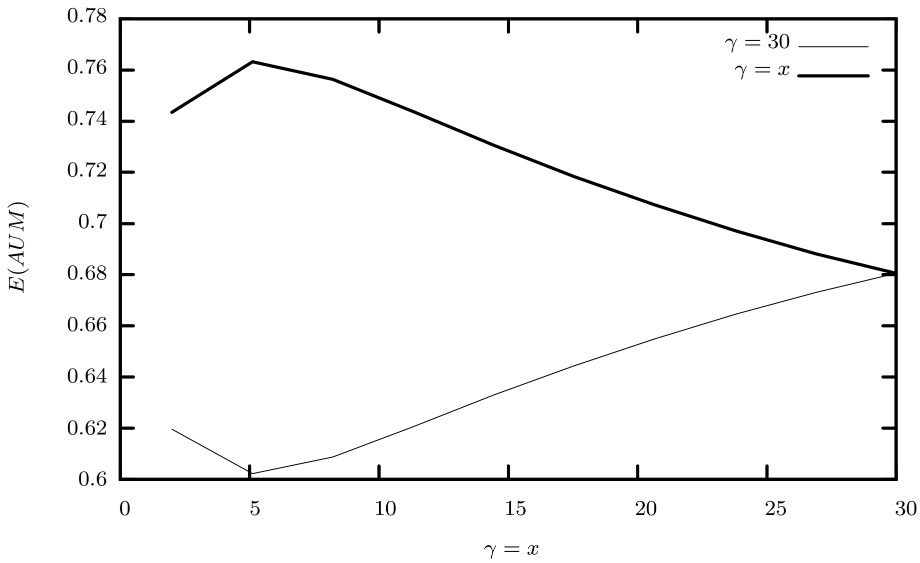

Figure 2: Long-run average AUM for a market with two companies. The coefficient of relative risk aversion of one company's strategy is plotted on the horizontal axis. The other company follows a CRRA strategy with coefficient $\gamma = 30$.

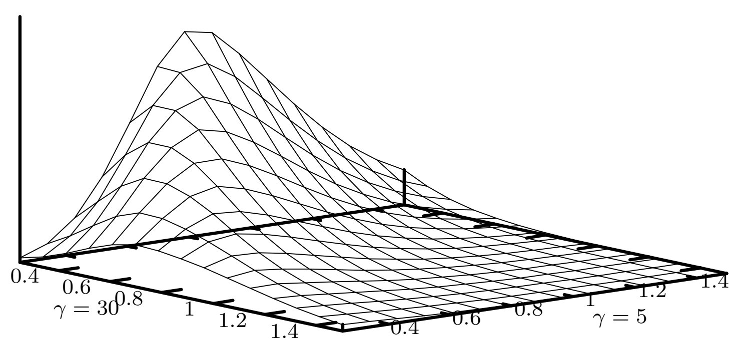

Figure 3: The distribution of AUM across two investment companies. One follows a strategy of $\gamma = 5$ while the follows the strategy $\gamma = 30$.

of relative risk aversion of 30. In other words, if the company's objective includes maximizing long-run average AUM, then it should not offer a fund whose strategy is to hold the market portfolio as measured by γ.

As can be noted, for the parameter values given there is an optimal level of riskiness that occurs at approximately $\gamma = 5$. For parameters of relative risk aversion greater than five, the increased riskiness of the portfolio and the consequent downside risk to AUM are outweighed by the fact that on average these portfolios earn a higher return. Since the mean with respect to the invariant distribution is the intertemporal average AUM, above $\gamma = 5$, this increase in risk leads to an increase in average AUM. However, as γ decreases below 5 the intertemporal averaging of high return outcomes is unable to completely offset the increased riskiness of the fund's portfolio.

4 Conclusion

It has been shown that the inclusion of agents who base decisions on recent past history and invest in mutual funds can lead to a non-degenerate distribution of asset portfolios, even in the long-run. Thus it is the case that the inclusion of investors who make investment decisions using past returns will cause the market selection hypothesis to not hold.

These results have have as yet unexplained implications for asset prices. Prices will certainly vary over the AUM distribution, and since prices are a function of the current AUM distribution they will have a trend. Exploring this relationship will be left to another paper.

Although I have used a particular investment rule for the investor and a particular interpretation of heterogeneous mutual fund behavior, the model constructed here is more general. It was constructed to understand the role of risk on observed AUM in a market with heterogeneous investors. In the future, it could be used to investigate questions of asset price dynamics, mutual fund regulation (including disclosure requirements) and fund fee structures.

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